

$$\Rightarrow \frac{dP}{-Pz} = \frac{dq}{-az} = \frac{dz}{-(3Pz^2 + P + 2(a-z))} = -\frac{dz}{(az^2 + 1)}$$

$$= \frac{dz}{-(az^2 + 1)}$$

$$\Rightarrow \frac{dP}{Pz} = \frac{dz}{az} = \frac{dz}{3Pz^2 + P + 2(a-z)}$$

From the first two ratios, we get

$$\frac{dP}{P} = \frac{dz}{az}$$

integrating we get $\log a - \log P + \log K$

$$= \log PK$$

putting $z = KP$ in the given equation,

$$P(1 + KP^2) = KP(z-a) \Rightarrow 1 + KP^2 = K(z-a)$$

$$\Rightarrow KP^2 = K(z-a) - 1 \Rightarrow P = \sqrt{K(z-a) - 1}$$

$$\therefore z = KP = \sqrt{K(z-a) - 1}$$

Now $dz = P \frac{dz}{dP} dP$

$$\Rightarrow dz = \frac{K(z-a) - 1}{K} dP \int \frac{K(z-a) - 1}{K} dP$$

$$\Rightarrow dz = \sqrt{K(z-a) - 1} \left(\frac{1}{K} dP dP \right)$$

$$\Rightarrow \frac{Kdz}{\sqrt{K(z-a) - 1}} = dP dP$$

integrating, we get

$$AK(z-a) - A = (x + ky + b)^2 \Rightarrow AK(z-a) = (x + ky + b)^2 + A$$

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D2 - Mathematics

problem based on characteristics method

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Q.1:- Solve by characteristics method to the
equation $P(1+q^2) = 2xz$

Solution:- We have $P(x, y, z, P, Q)$

$$= P(1+q^2) - 2(1-q)z = 0$$

$$\frac{\partial F}{\partial x} = 0, \quad \frac{\partial F}{\partial y} = 0, \quad \frac{\partial F}{\partial z} = 2 - 2$$

$$\frac{\partial F}{\partial P} = 1+q^2, \quad \frac{\partial F}{\partial Q} = 2PQ - (1-q)$$

Hence characteristics equation are

$$\frac{dx}{1+q^2} = \frac{dy}{0+q(1-q)} = \frac{dz}{2PQ - (1-q)}$$

$$= \frac{dx}{-(1+q^2)} = \frac{dy}{-2PQ + (1-q)}$$